4

RR-88-57-ONR

AD-A203 835

DTIC FILE COPY

SPECIFYING OPTIMUM EXAMINEES FOR ITEM PARAMETER ESTIMATION IN ITEM RESPONSE THEORY

Martha L. Stocking

This research was sponsored in part by the
Cognitive Science Program
Cognitive and Neural Sciences Division
Office of Naval Research, under
Contract No. N00014-83-K-0457

Contract Authority Identification No.
NR 150-520

Frederic M. Lord, Principal Investigator

Educational Testing Service
Princeton, New Jersey

October 1988

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
Unclassified

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
RR-88-57-ONR

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
Educational Testing Service

6b. OFFICE SYMBOL *(If applicable)*

7a. NAME OF MONITORING ORGANIZATION
Cognitive Science Program, Office of Naval Research (1142PT), 800 North Quincy Street

6c. ADDRESS *(City, State, and ZIP Code)*
Princeton, NJ 08541

7b. ADDRESS *(City, State, and ZIP Code)*
Arlington, VA 22217-5000

8a. NAME OF FUNDING / SPONSORING ORGANIZATION

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
N00014-83-K-0457

8c. ADDRESS *(City, State, and ZIP Code)*

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO	PROJECT NO	TASK NO	WORK UNIT ACCESSION NO
61153N	RR04204	RR04204-01	NR150-520

11. TITLE *(Include Security Classification)*
Specifying Optimum Examinees for Item Parameter Estimation in Item Response Theory (Unclassified)

12. PERSONAL AUTHOR(S)
Martha L. Stocking

13a. TYPE OF REPORT
Technical

13b. TIME COVERED
FROM ________ TO ______

14. DATE OF REPORT *(Year, Month, Day)*
October 1988

15. PAGE COUNT
31

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP
05	10	

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Item response theory, Information functions.
IRT item parameters
Optimum examinees

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

Information functions are used to find the optimum ability levels and maximum contributions to information for estimating item parameters in three commonly used logistic item response models. For the three and two parameter logistic models, examinees who contribute maximally to the estimation of item difficulty contribute little to the estimation of item discrimination. This suggests that in applications that depend heavily upon the veracity of individual item parameter estimates (e.g. adaptive testing or test construction), better item calibration results may be obtained (for fixed sample sizes) from examinee calibration samples in which ability is widely dispersed.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL
Dr. Charles E. Davis

22b. TELEPHONE *(Include Area Code)*
202-696-4046

22c. OFFICE SYMBOL
ONR 1142CS

DD Form 1473, JUN 86 *Previous editions are obsolete.*

S/N 0102-LF-014-6603

SECURITY CLASSIFICATION OF THIS PAGE

Unclassified

Specifying Optimum Examinees for Item Parameter Estimation in Item Response Theory

Martha L. Stocking

Educational Testing Service

Princeton, New Jersey

October 1988

Abstract

Information functions are used to find the optimum ability levels and maximum contributions to information for estimating item parameters in three commonly used logistic item response models. For the three and two parameter logistic models, examinees who contribute maximally to the estimation of item difficulty contribute little to the estimation of item discrimination. This suggests that in applications that depend heavily upon the veracity of individual item parameter estimates (e.g. adaptive testing or test construction), better item calibration results may be obtained (for fixed sample sizes) from examinee calibration samples in which ability is widely dispersed.

Keywords: Item response theory
IRT item parameters
Optimum examinees
Information functions

Introduction

The success of applications in Item Response Theory (IRT) depends upon the accuracy with which individual item parameters can be estimated. This dependence is especially important for those applications that depend heavily upon the veracity of individual item parameter estimates in contexts perhaps unrelated to the calibration setting in which estimates were obtained. Two recent examples of such applications are adaptive testing (see, for example, Lord (1980, chap. 10); Stocking (1988)) and IRT-based test development (see, for example, van der Linden & Boekkooi-Timminga (1987)).

Typically, calibration samples are selected with an eye to convenience as random subsamples from a larger set of data (see, for example, Cook, Petersen, & Stocking (1983)). It may be possible to obtain better, i.e., more accurate, item parameter estimates if the planning for calibration sample selection explicitly includes considerations of the accuracy of the resulting item parameter estimates. This idea was suggested, based on the analogy between item parameter estimation and estimation problems in regression, 20 years ago by Lord (1968). More recently, Wingersky and Lord (1984) provide theoretical support.

One approach to issues of the accuracy of item parameter estimation considers the asymptotic expected covariance matrix. Lord (1980, p. 191) presents formulas from which this matrix can be derived for the three-parameter logistic (3PL) model in the finite sample case with known abilities. Thissen and Wainer (1982) consider the properties of the asymptotic covariance matrix for item parameters when a particular distribution of ability is assumed in the calibration example. Lord and Wingersky (1985) consider the asymptotic properties of the covariance matrix for the 3PL model when both item parameters and abilities are estimated jointly by maximum likelihood.

In this paper we explore the relationship between examinee ability and the accuracy of maximum likelihood item parameter estimation in terms of the expected (Fisher) information (Kendall & Stuart, 1979, p. 10). The advantage to this approach is that it is possible to study directly the relationship between examinee ability and contribution to information in some detail and with some surprising results. This theoretical examination depends upon the assumption that examinee abilities, as well as some item parameters, are known. However, the results can be translated into general guidelines for situations in which samples can be selected on the basis of some observed score for the purpose of obtaining accurate

parameter estimates for a collection of items whose properties are imperfectly known.

The Theory

Suppose the probablility of a correct response to an item is specified by a logistic function, either one parameter logistic (1PL), two parameter logistic (2PL) or three parameter logistic (3PL). Suppose further that estimates of the item parameters are obtained by maximum likelihood. If the model fits the data, and true abilities are known, then formulas exist for the amount of information in the sample for estimating item parameters (Lord, 1980, eqs. 12-8 through 12-13)).

Of course, the model never fits any set of real data, and true abilities are never known. Therefore, the information computed from the Lord formulas is an estimate of the maximum possible information, and will be larger than what can be realistically obtained (Mislevy & Sheehan, 1987). Nevertheless, an examination of what the theory predicts may be useful in planning a calibration sample to estimate item parameters accurately.

If the probability of a correct response by examinee a to an item is P_a, the log of the likelihood of observed responses to the item for N examinees is

$$\ell = \sum_{a=1}^{N} [u_a \ell n P_a + (1 - u_a) \ell n Q_a] \quad ,$$

where $u_a = 0$ if the response in incorrect, $u_a = 1$ if the response is correct, and $Q_a = 1 - P_a$. The maximum likelihood estimates of each item parameter χ are located at a point where the partial derivatives of the log likelihood are zero. The expected second partial derivatives of the log likelihood can be computed at this point. (The inverse of the negative of this matrix is the asymptotic variance/covariance matrix used by Thissen and Wainer (1982).) The negative of a diagonal element of the matrix of expected second partial derivatives is referred to as the information in the sample for estimating an item parameter. For any item parameter χ, this information function has the form (Lord, 1980, equations 12-8 to 12-13)

$$I_{\chi\chi} = \sum_{a=1}^{N} \frac{1}{P_a Q_a} \left(\frac{\partial P_a}{\partial \chi} \right)^2 = \sum_{a=1}^{N} i_{\chi\chi_a} \quad .$$

The information is composed of individual additive contributions from each examinee,

$$i_{\chi\chi} = \frac{1}{PQ} \left(\frac{\partial P}{\partial \chi} \right)^2 \qquad (1)$$

where the subscript a has been dropped for convenience. By examining the properties of an individual contribution as a function of ability, we can determine what values of ability provide the most (and least) information for estimating an item parameter, assuming other item parameters are fixed.

Results

The 3PL Model

The 3PL item response function is

$$P = c + \frac{1 - c}{1 + e^{-Da(\theta - b)}} \quad .$$

where

a is a function of the slope of the item response function in the neighborhood of the item difficulty;

b characterizes the difficulty or location of the item on the ability continuum;

c is the lower bound of probabilities of correct response, even from low ability examinees;

θ is examinee ability;

D is a scaling constant commonly used with the value of 1.7.

The partial derivatives of P with respect to the item parameters are given by Lord (1980, Equation 12-2). Substituting the derivative with respect to c into Equation 1 gives an examinee's contribution to the information for estimating c as

$$i_{cc} = \frac{1}{(1 - c)^2} \frac{Q}{P} \quad . \tag{2}$$

Values of ability that give local minima and maxima of Equation 2 are found where

$$\frac{\partial i_{cc}}{\partial \theta} = \frac{-Da}{(1 - c)^3} \frac{1}{P^2} Q(P - c) = 0 \quad .$$

This derivative is zero when $Q = 0$, in which case $\theta = +\infty$, and i_{cc} is zero. It is also zero when $P = c$, in which case $\theta = -\infty$ and $i_{cc} = \frac{1}{(1 - c)c}$. This latter value forms an upper asymptote to the amount of information from all ability levels. The higher the value of c, the more examinees, even optimal examinees, are required for estimation of c.

The information for estimating b is obtained by substituting the derivative of P with respect to b into Equation 1, giving

$$i_{bb} = \frac{D^2 a^2}{(1 - c)^2} (P - c)^2 \frac{Q}{P} \quad , \tag{3}$$

with partial derivative

$$\frac{\partial i_{bb}}{\partial \theta} = \frac{D^3 a^3}{(1 - c)^3} \frac{Q(P - c)^2}{P^2} [-2P^2 + P + c] \quad . \tag{4}$$

A root of this derivative is found where $Q = 0$, in which case $\theta = +\infty$ and $i_{bb} = 0$. A second root is found when $P = c$, where $\theta = -\infty$ and $i_{bb} = 0$. Examinees with abilities far away from the item's location are useless for estimating that location. A final root of this derivative is found where $-2P^2 + P + c = 0$. The root of this latter quantity, P^*, where $c \le P^* \le 1$, is $P^* = \frac{1 + \sqrt{1 + 8c}}{4}$. The optimal ability θ^* is then

$$\theta^* = b + \frac{1}{Da} \ell n\left(\frac{P^* - c}{1 - P^*} \right) \tag{5}$$

and

$$i_{bb} = \frac{D^2 a^2}{(1 - c)^2} (P^* - c)^2 \frac{Q^*}{P^*} \quad , \tag{6}$$

where $Q^* = 1 - P^*$.

The optimal ability level is at $\theta^* = b$ when $c = 0$. When $0 < c < 1$, the optimal ability level is greater than b by an amount that depends upon both a and c. For a fixed value of c, higher values of a mean that the optimum location is closer to but still greater than b. Since P^* is a function of c alone, the maximum amount of information from an examinee for estimating b, Equation 6, depends only on a and c, and for fixed c, is proportional to the square of a.

By analagous computations, the information from each examinee (Equation 1) for estimating a for the 3PL is

$$i_{aa} = \frac{D^2}{(1 - c)^2} (\theta - b)^2 (P - c)^2 \frac{Q}{P} \quad , \tag{7}$$

with partial derivative

$$\frac{\partial i_{aa}}{\partial \theta} = \frac{D^2}{(1 - c)^2} \frac{(\theta - b)(P - c)^2 Q}{P^2} [(\theta - b) \frac{Da}{(1 - c)} (-2P^2 + P + c) + 2P] \quad (8)$$

Three roots of Equation 8 are at $\theta = b$, $\theta = -\infty$, and $\theta = +\infty$. For all three of these roots, $i_{aa} = 0$. Low and high ability examinees, as well as examinees whose abilities may be close to optimum for estimating b are of no use in the estimation of a. The location of optimal abilities is found from the last factor in Equation 8. This expression is zero when the optimal ability

$$\theta^{**} = b + \frac{2(1 - c)P^{**}}{Da(2P^{**2} - P^{**} - c)} \quad . \quad (9)$$

The optimal ability is not given explicitly by this expression, but values of θ^{**} can be found by numerical methods. The top panel of Figure 1 shows P plotted as a function of θ - b, from Equation 9. When $P = c$, $\theta - b = -\frac{1}{Da}$, and when $P = 1$, $\theta - b = \frac{2}{Da}$. The plot suggests that these values form upper and lower bounds respectively for two regions in which the optimal ability might be found: $\theta < b - \frac{1}{Da}$ and $\theta > b + \frac{2}{Da}$.

Insert Figure 1 about here

Numerical Examples

Equations 2, 3, and 7 are plotted as functions of ability in the top panel of Figure 2 for an item with $a = 1$, $b = 0$, and $c = .2$. Additive contributions to information for estimating a or b from examinees at various ability levels must be read from the right-hand scale; those for estimating c must be read from the left-hand scale. Figure 2 demonstrates graphically what we have learned analytically: 1) the contribution of examinees to the information available for estimating item difficulty is asymetric around b, with higher ability examinees contributing more information; 2) examinees who are most informative in the estimation of item difficulty are of little use in estimating item discrimination; 3) examinees who do contribute to the estimation of discrimination are asymetrically distributed around the item difficulty; and 4) only low ability examinees contribute much information for the estimation of c, and there is a limit to this contribution.

Insert Figure 2 about here

The effects of different values of a, for fixed b and c, on the additive contribution of examinees to information for estimating a are shown in the top left panel of Figure 3. Each examinee contributes substantially more information to the estimation of a if the value of a is low rather than high. More lower ability examinees than higher ability examinees are required to obtain a given amount of information for the estimation of a, regardless of the value of a, since lower ability examinees contribute less information than higher ability examinees.

Insert Figure 3 about here

The middle left panel of Figure 3 shows the effects of different values of b, for fixed a and c, on the additive contribution of examinees to information for estimating b. Changing b shifts the location of optimal examinees, but not the maximum contribution. The effects of different c's on the additive contribution for estimating c, for fixed a and b, are shown in the bottom left panel of Figure 3. For lower ability levels, the higher the guessing parameter, the less the additive contribution for each examinee; more examinees are required to obtain a given amount of information about c. Regardless of the value of c, higher ability examinees do

not contribute much. However, they contribute more if c is higher than if c is lower.

Numerical values of optimal abilities and maximal contributions to information (in parentheses) are presented for typical values of a and b for $c = .2$ in Table 1. Optimal abilities and maximal contributions for b come directly from Equations 5 and 6. Those for a come from the application of numerical methods to Equations 9 and 7.

Insert Table 1 about here

The 2PL and 1PL Models

The 2PL item response function can be considered as a special case of the 3PL with $c = 0$. In this case,

$$P = \frac{1}{1 + e^{-Da(\theta-b)}} \quad .$$

The 1PL item response function is also a special case of the 3PL with $c = 0$ and constant a. The expression for P is identical to that for the 2PL immediately above, but the item discrimination is constant for all items.

The information for estimating b for the 2PL is algebraically the same as for the 1PL:

$$i_{bb} = D^2a^2PQ \quad , \tag{10}$$

with derivative with respect to ability

$$\frac{\partial i_{bb}}{\partial \theta} = D^3a^3PQ(1 - 2P) \quad .$$

These are obtained form Equations 3 and 4 for the 3PL by setting $c = 0$. Examinees with abilities of $\pm\infty$ contribute nothing to the estimation of b in these models. Examinees with $\theta^* = b$ ($P^* = .5$) contribute the maximum amount of information, $.25D^2a^2$. The additive contribution from each examinee depends upon the square of the discrimination of the item.

For the 2PL model, the information for estimating a is

$$i_{aa} = D^2(\theta - b)^2PQ \tag{11}$$

with derivative

$$\frac{\partial i_{aa}}{\partial \theta} = D^2(\theta - b)PQ[Da(\theta - b)(1 - 2P) + 2] \quad ,$$

again obtained from the 3PL results (Equations 7 and 8) by setting $c = 0$. As with the 3PL, examinees with low and high abilities, as well as examinees whose abilities are optimal for estimating b are useless for estimating a. The location of optimal abilities is found by setting $Da(\theta - b)(1 - 2P) + 2 = 0$ in which case the optimal ability θ^{**} is

$$\theta^{**} = b + \frac{2}{Da(2P - 1)} \quad . \tag{12}$$

The bottom panel of Figure 1 shows P plotted as a function of $\theta - b$ for the 2PL in Equation 12. When $P = 0$, $\theta - b = -\frac{2}{Da}$, and when $P = 1$, $\theta - b = \frac{2}{Da}$. As with the 3PL, there are two regions in which θ^{**} might be found: $\theta < b - \frac{2}{Da}$ and $\theta > b + \frac{2}{Da}$. In contrast to the 3PL, these regions are symmetric around $\theta - b = 0$.

The additive contribution to estimating a for optimal examinees is

$$i_{aa} = .25D^2(\theta^{**} - b)^2 - \frac{1}{a^2} \quad . \tag{13}$$

Both terms in Equation 13 decrease for higher values of a since the optimal θ is closer to the difficulty for increased a, given fixed b. In contrast with the 3PL, the contributions of examinees whose true abilities are symmetric around the item difficulty are identical.

Numerical Examples

Equation 10 for the 2PL and 1PL and Equation 11 for the 2PL are plotted as fuctions of ability in the bottom panel of Figure 2 for an item with $a = 1$ and $b = 0$. Additive contributions to information for estimating a (for fixed b) or b (for fixed a) are higher in models that do not contain a guessing parameter. This is seen by comparing the two panels in Figure 2.

The effects of different a values (with fixed b) on the additive contributions for estimating a in the 2PL (Equation 11) are shown in the top right panel of Figure 3. Examinees symmetrically located around the item difficulty contribute equally to information. As in the 3PL (top left panel), each examinee contributes more to information for estimating a when a is low. In contrast to the 3PL, examinee's additive contributions are higher when the model does not contain a guessing parameter (compare the two top panels of Figure 3). Equation 10 for the 2PL with fixed a or the 1PL is plotted for different values of b in the middle right

panel of Figure 3. As with the 3PL (middle left panel), changing values of b shifts the location of optimal examinees but not the maximum contribution to information for estimating item difficulty.

Numerical values of optimal abilities and maximum contributions to information (in parentheses) are presented for typical values of a and b for the 2PL and 1PL in Table 2. Optimal abilities and maximum contributions for estimating b come directly from Equation 10 and its consequences. Comparable data for information for estimating a for the 2PL comes from the application of numerical methods to Equation 12 and Equation 13.

Insert Table 2 about here

Discussion

The focus of this paper has been to explore the contribution of examinees with different true abilities to the expected information available for estimating item parameters. This is not the same as an exploration of the accuracy with which item parameters may be estimated, but has the advantage of directly suggesting strategies of calibration sample selection. In terms of making statements about the accuracy with which item parameters are estimated, the analyses presented here are immediately relevant to the problem of

estimating a single item parameter, conditional on other item parameters being fixed at their true values. However, statements about the accuracy of item parameter estimates are usually made in a broader context in which all three item parameters must be estimated and a particular distribution of true ability may be assumed. Such considerations then take into account the covariances among the item parameters, and depend upon the assumed distribution of ability. The results need not hold for different ability distributions.

As an example, the results presented here show that in both the 2PL and 3PL, optimal examinees contribute more information for estimating item discrimination when that discrimination is low. This cannot be interpreted to mean that low a's are necessarily more accurately estimated than high a's, for a fixed sample size or a fixed distribution of ability. Thissen and Wainer (1982), using the expected asymptotic variance/covariance matrix and assuming a normal distribution of ability, show that, for fixed b, the asymptotic standard error of a decreases with a for the 2PL. However, for fixed b in the 3PL model, the asymptotic error of a is larger for both small and large values of a than it is for more moderate values of a. These results depend on the assumed distribution of ability as well as the covariances among the item parameters.

As an illustration of this phenomenon, Figure 4 shows the Thissen and Wainer asymptotic standard error of b plotted against b for the 3PL (solid curve). This is a partial reproduction of Thissen and Wainer's Figure 1, with $a = 1.5$, $c = 0.$, and a standard normal calibration sample of $N = 2500$. The results presented here suggest that a calibration sample with a larger spread of abilities will improve the accuracy of the estimation of high and low difficulties. Figure 4 shows the same information for the same item when a calibration sample of the same size is drawn from a normal distribution of ability with a variance of 4 (dashed line). Although the sample si~e is the same, the accuracy of estimation is much improved for the more extreme difficulties.

Insert Figure 4 about here

Given the knowledge of the location of optimal examinees for estimating the parameters for a single item in three different item response models presented here, what suggestions can we make for calibration sample selection that would aid practitioners who must jointly estimate the item parameters for many items simultaneously? If the practitioner chooses the 1PL as the appropriate model, then examineees whose true ability is equal to item difficulty are most

informative. If a collection of items to be calibrated is thought to have a broad spread of difficulties, based perhaps on the conventional proportions correct, then the distribution of true abilities in the calibration sample should also be broad. Such a sample could possibly be selected based on some available observed auxiliary information. If the range of abilities is too small, the sample will provide information only for middle difficulty items; little information will be provided to estimate the difficulty for easy and hard items.

If the 2PL is the appropriate model, estimation of both a and b requires a wider range of true abilities than for the 1PL. This is so because only examinees with ability not equal to b are informative in the estimation of a. If the range of abilities is too small, information for the estimation of difficulties for middle level items may be provided, but information for estimating their discrimination may not be. Information for the estimation of discriminations for easy and hard items may be provided, but information for estimating their difficulties may not be.

If the 3PL is the most appropriate model, only abilities well below the item difficulty are informative about c. Abilities below and above the difficulty are most informative about a and abilities slightly above the difficulty are most informative about b. Even if

all items are of equal difficulty, a normal ditribution of abilities centered slightly above the item difficulty may not provide much information for the estimation of all parameters simultaneously. If the items have a spread of difficulties, better results may be obtained by sampling all ability levels equally. Wingersky and Lord (1984) show that when item and ability parameters are estimated simultaneously, a sample of abilities drawn from a uniform distribution produces standard errors nearly as small as a sample of abilities four times as large drawn from a bell-shaped distribution.

Calibration samples, particularly for the more complex models, typically consist of several thousand examinees. Depending upon the nature of the collection of items to be calibrated, which can be roughly assessed through the use of conventional item statistics, such samples, although large, may not prove useful for estimating the paramters of all items. If the success of a particular application of IRT depends heavily on the veracity of item level data, it seems worthwhile to consider selecting more informative samples.

References

Cook, L. L., Petersen, N. J., & Stocking, M. L. (1983). IRT versus conventional equating methods: A comparative study of scale stability. Journal of Educational Statistics, 8, 136-156.

Kendall, M., & Stuart, A. (1979). The advanced theory of statistics, Volume 2, Inference and Relationship. New York: Macmillan.

Lord, F. M. (1968). An analysis of the Verbal Scholastic Aptitude Test using Birnbaum's three-parameter logistic model. Educational and Psychological Measurement, 28, 989-1020.

Lord, F. M. (1980). Applications of item response theory to practical testing problems. Hillsdale, NJ: Erlbaum.

Lord, F. M. (1983). Statistical bias in maximum likelihood estimators of item parameters. Psychometrika, 48, 425-435.

Lord, F. M., & Wingersky, M. S. (1985). Sampling variances and covariances of parameter estimates in item response theory. In D. J. Weiss (Ed.), Proceedings of the 1982 IRT/CAT Conference (pp. 69-88). Minneapolis, MN: CAT Laboratory, Department of Psychology, University of Minnesota.

Mislevy, R. J., & Sheehan, K. M. (1988). The information matrix in latent variable model. Research Report 88-24. Princeton, NJ: Educational Testing Service.

Stocking, M. L. (1988). Scale drift in on-line calibration. Report 88-24. Princeton, NJ: Educational Testing Service.

Thissen, D., & Wainer, H. (1982). Some standard errors in item response theory. Psychometrika, 47, 397-412.

Wingersky, M. S., & Lord, F. M. (1984). An investigation of methods for reducing sampling error in certain IRT procedures. Applied Psychological Measurement, 8, 347-364.

van der Linden, W. J., & Boekkooi-Timminga, E. (1987). A maximum model for test design with practical constraints. Research Report 87-10. Princeton, NJ: Educational Testing Service.

Acknowledgment

This work was supported by Contract No. N00014-83-C-0457, project designation NR 150-520, from Cognitive Science Program, Cognitive and Neural Sciences Division, Office of Naval Research and Educational Testing Service through the Program Research Planning Council. Reproduction in whole or in part is permitted for any purpose of the United States Government. The author wishes to acknowledge the invaluable assistance of Maxine B. Kingston in carrying out this study, and to thank Dr. Charles Lewis for his many insightful comments on earlier drafts of this paper.

Table 1

Optimum Abilities and Maximum Contributions (in Parentheses) to Information for Estimating a or b Assuming Fixed Values for the Other Parameters for the 3PL Model with c = .2

a	b	lower optimal θ for a	upper optimal θ for a	optimal θ for b
.5	-2	-3.96	.86	-1.69
(i_{aa} = .58, 1.38)	-1	-2.96	1.86	-.69
	0	-1.96	2.86	.31
(i_{bb} = .12)	1	-.96	3.86	1.31
	2	.04	4.86	2.31
1.0	-2	-2.98	-.57	-1.84
(i_{aa} = .14, .35)	-1	-1.98	.43	-.84
	0	-.98	1.43	.16
(i_{bb} = .49)	1	.02	2.43	1.16
	2	1.02	3.43	2.16
1.5	-2	-2.65	-1.05	-1.90
(i_{aa} = .06, .15)	-1	-1.65	-.05	-.90
	0	-.65	.95	.10
(i_{bb} = 1.11)	1	.35	1.95	1.10
	2	1.35	2.95	2.10
2.0	-2	-2.49	-1.28	-1.92
(i_{aa} = .04, .09)	-1	-1.49	-.28	-.92
	0	-.49	.72	.08
(i_{bb} = 1.97)	1	.51	1.72	1.08
	2	1.51	2.72	2.08
2.5	-2	-2.39	-1.43	-1.94
(i_{aa} = .02, .06)	-1	-1.39	-.43	-.94
	0	-.39	.57	06
(i_{bb} = 3.08)	1	.61	1.57	1.06
	2	1.61	2.57	2.06

Table 2

Optimum Abilities and Maximum Contributions (in Parentheses) to Information for Estimating a or b Assuming Fixed Values for the Other Parameters for the 2PL and 1PL Models with c = 0

a	b	lower optimal θ for a	upper optimal θ for a	optimal θ for b
.5	-2	-4.82	.82	-2.00
(i_{aa} = 1.76, 1.76)	-1	-3.82	1.82	-1.00
	0	-2.82	2.82	.00
(i_{bb} = .18)	1	-1.82	3.82	1.00
	2	-.82	4.82	2.00
1.0	-2	-3.41	-.59	-2.00
(i_{aa} = .44, .44)	-1	-2.41	.41	-1.00
	0	-1.41	1.41	.00
(i_{bb} = .72)	1	-.41	2.41	1.00
	2	.59	3.41	2.00
1.5	-2	-2.94	-1.06	-2.00
(i_{aa} = .20, .20)	-1	-1.94	-.06	-1.00
	0	-.94	.94	.00
(i_{bb} = 1.63)	1	.06	1.94	1.00
	2	1.06	2.94	2.00
2.0	-2	-2.70	-1.30	-2.00
(i_{aa} = .11, .11)	-1	-1.70	-.30	-1.00
	0	-.70	.70	.00
(i_{bb} = 2.90)	1	.30	1.70	1.00
	2	1.30	2.70	2.00
2.5	-2	-2.56	-1.44	-2.00
(i_{aa} = .07, .07)	-1	-1.56	-.44	-1.00
	0	-.56	.56	.00
(i_{bb} = 4.53)	1	.44	1.56	1.00
	2	1.44	2.56	2.00

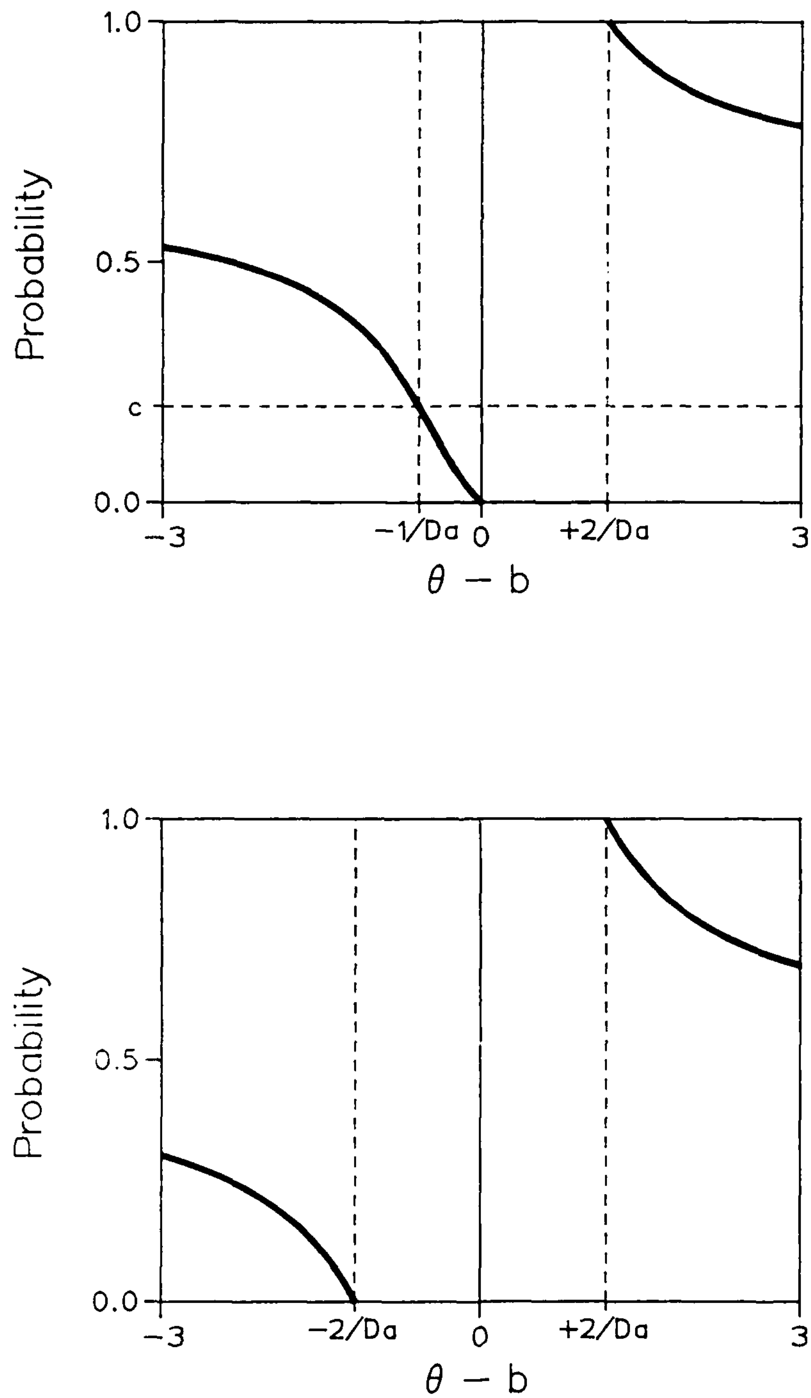
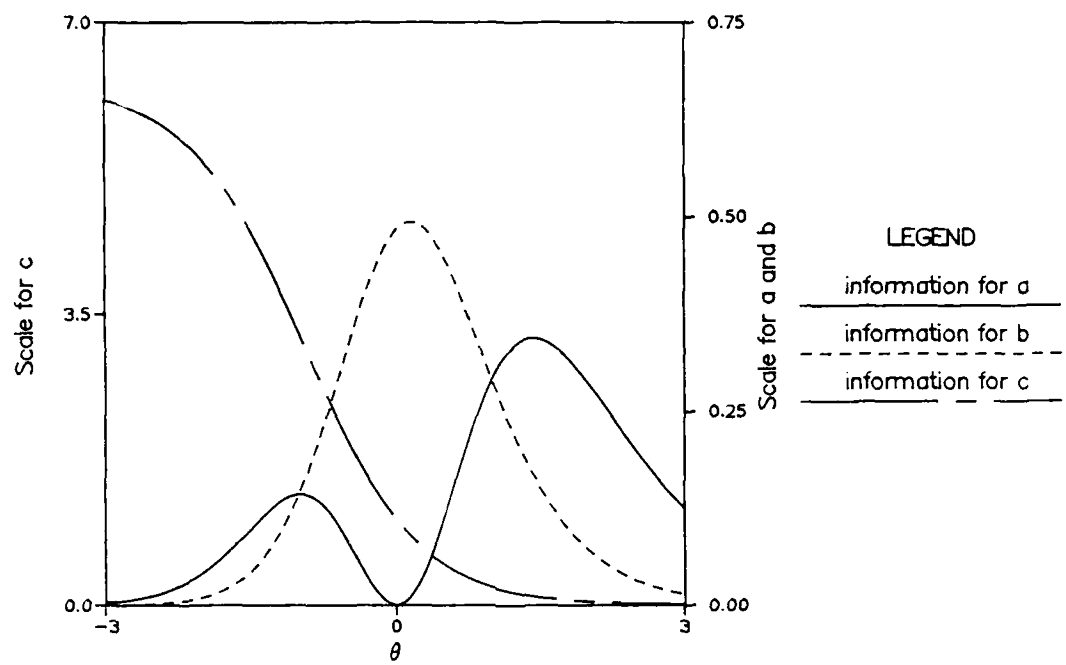


Figure 1. The top panel shows P as a function of $\theta - b$ from Equation 9 for the 3PL (solid curves). The bottom panel shows P as a function of $\theta - b$ from Equation 12 for the 2PL (solid curves).



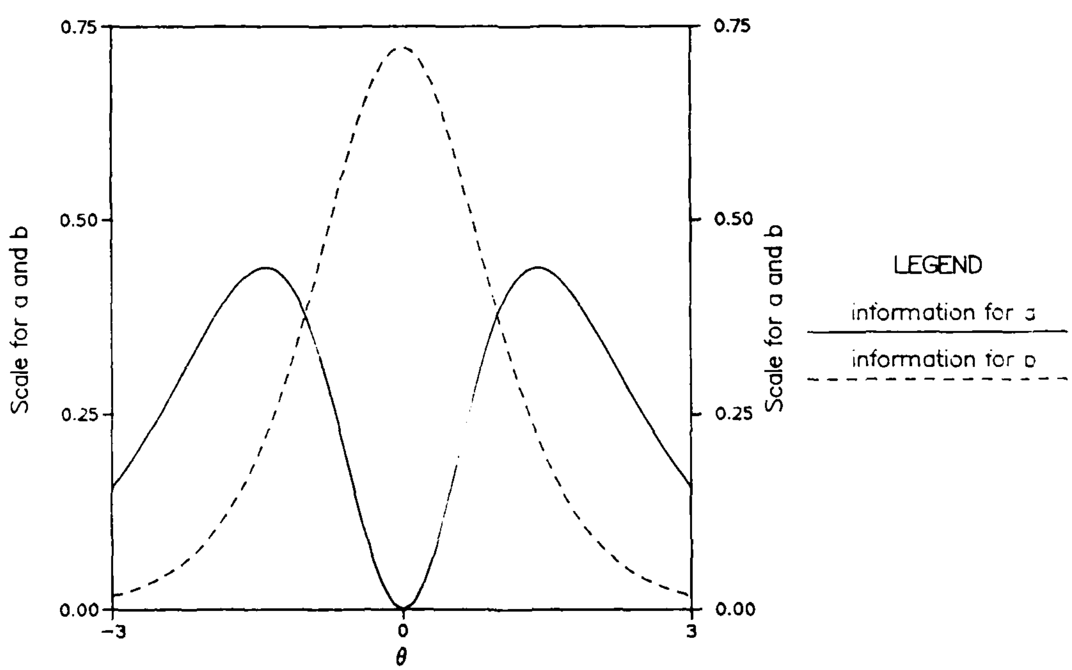


Figure 2. Contributions to information for estimating item parameters as functions of ability for the 3PL (top; a=1, b=0, c=.2), the 2PL and 1PL (bottom; a=1, b=0).

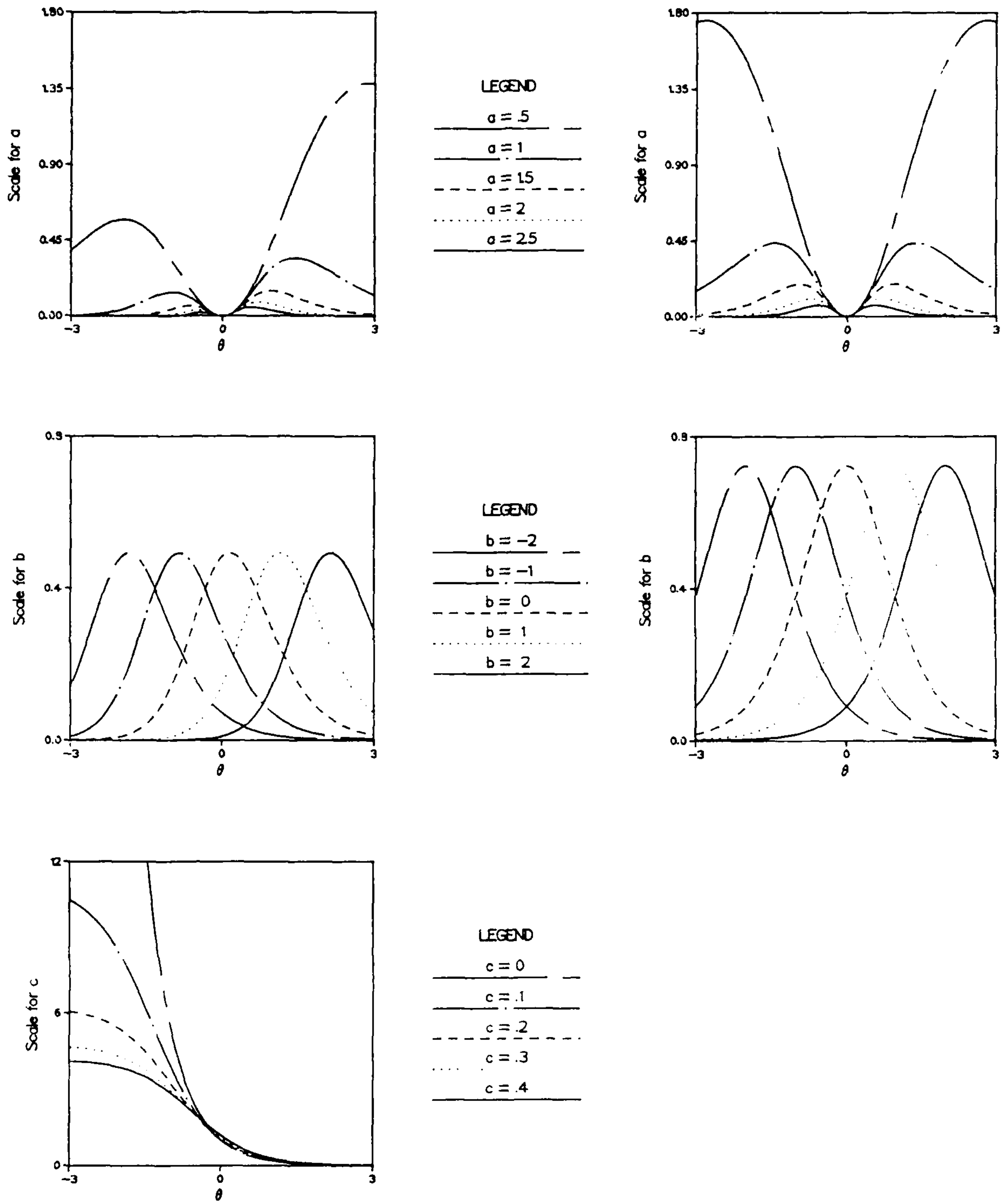


Figure 3. Contributions to information for estimating item parameters for the 3PL (left) and 2PL and 1PL (right). The effects of varying a conditional on fixed b=0 (and c=.2 for the 3PL) are shown in the top row. The effects of varying b for fixed a=1 (and c=.2 for the 3PL) are shown in the middle row. The effects of varying c for the 3PL conditional on a=1 and b=0 are shown in the bottom row.

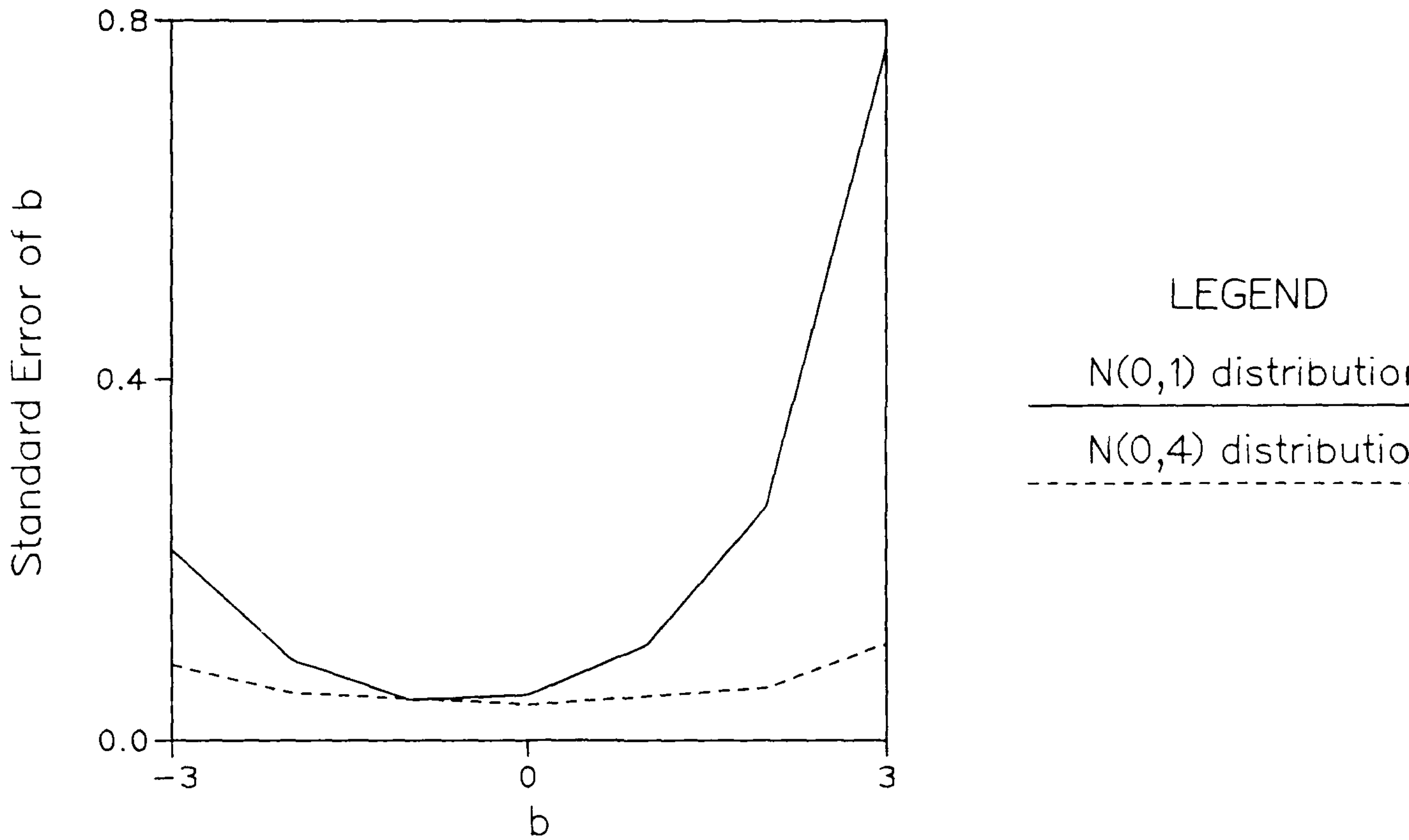


Figure 4. Standard errors of estimated item difficulties for the 3PL model, a=1.5, c=0. A calibration sample of N=2,500 is drawn from an N(0,1) distribution (solid) and from an N(0,4) distribution (dotted).

Dr. Terry Ackerman
American College Testing Programs
P.O. Box 168
Iowa City, IA 52243

Dr. Robert Ahlers
Code N711
Human Factors Laboratory
Naval Training Systems Center
Orlando, FL 32813

Dr. James Algina
1403 Norman Hall
University of Florida
Gainesville, FL 32605

Dr. Erling B. Andersen
Department of Statistics
Studiestraede 6
1455 Copenhagen
DENMARK

Dr. Eva L. Baker
UCLA Center for the Study
of Evaluation
145 Moore Hall
University of California
Los Angeles, CA 90024

Dr. Isaac Bejar
Mail Stop: 10-R
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Menucha Birenbaum
School of Education
Tel Aviv University
Ramat Aviv 69978
ISRAEL

Dr. Arthur S. Blaiwes
Code N712
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Bruce Bloxom
Defense Manpower Data Center
550 Camino El Estero,
Suite 200
Monterey, CA 93943-3231

Dr. R. Darrell Bock
University of Chicago
NORC
6030 South Ellis
Chicago, IL 60637

Cdt. Arnold Bohrer
Sectie Psychologisch Onderzoek
Rekruterings-En Selectiecentrum
Kwartier Koningen Astrid
Bruijnstraat
1120 Brussels, BELGIUM

Dr. Robert Breaux
Code 7B
Naval Training Systems Center
Orlando, FL 32813-7100

Dr. Robert Brennan
American College Testing
Programs
P. O. Box 168
Iowa City, IA 52243

Dr. James Carlson
American College Testing
Program
P.O. Box 168
Iowa City, IA 52243

Dr. John B. Carroll
409 Elliott Rd., North
Chapel Hill, NC 27514

Dr. Robert M. Carroll
Chief of Naval Operations
OP-01B2
Washington, DC 20350

Dr. Raymond E. Christal
UES LAMP Science Advisor
AFHRL/MOEL
Brooks AFB, TX 78235

Dr. Norman Cliff
Department of Psychology
Univ. of So. California
Los Angeles, CA 90089-1061

Director,
Manpower Support and
Readiness Program
Center for Naval Analysis
2000 North Beauregard Street
Alexandria, VA 22311

Dr. Stanley Collyer
Office of Naval Technology
Code 222
800 N. Quincy Street
Arlington, VA 22217-5000

Dr. Hans F. Crombag
Faculty of Law
University of Limburg
P.O. Box 616
Maastricht
The NETHERLANDS 6200 MD

Dr. Timothy Davey
Educational Testing Service
Princeton, NJ 08541

Dr. C. M. Dayton
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Ralph J. DeAyala
Measurement, Statistics,
and Evaluation
Benjamin Bldg., Rm. 4112
University of Maryland
College Park, MD 20742

Dr. Dattprasad Divgi
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Hei-Ki Dong
Bell Communications Research
6 Corporate Place
PYA-1K226
Piscataway, NJ 08854

Dr. Fritz Drasgow
University of Illinois
Department of Psychology
603 E. Daniel St.
Champaign, IL 61820

Defense Technical
Information Center
Cameron Station, Bldg 5
Alexandria, VA 22314
Attn: TC
(12 Copies)

Dr. Stephen Dunbar
224B Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. James A. Earles
Air Force Human Resources Lab
Brooks AFB, TX 78235

Dr. Kent Eaton
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. John M. Eddins
University of Illinois
252 Engineering Research
Laboratory
103 South Mathews Street
Urbana, IL 61801

Dr. Susan Embretson
University of Kansas
Psychology Department
426 Fraser
Lawrence, KS 66045

Dr. George Englehard, Jr.
Division of Educational Studies
Emory University
210 Fishburne Bldg.
Atlanta, GA 30322

Dr. Benjamin A. Fairbank
Performance Metrics, Inc.
5825 Callaghan
Suite 225
San Antonio, TX 78228

Dr. P-A. Federico
Code 51
NPRDC
San Diego, CA 92152-6800

Dr. Leonard Feldt
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Richard L. Ferguson
American College Testing
P.O. Box 168
Iowa City, IA 52243

Dr. Gerhard Fischer
Liebiggasse 5/3
A 1010 Vienna
AUSTRIA

Dr. Myron Fischl
U.S. Army Headquarters
DAPE-MRR
The Pentagon
Washington, DC 20310-0300

Prof. Donald Fitzgerald
University of New England
Department of Psychology
Armidale, New South Wales 2351
AUSTRALIA

Mr. Paul Foley
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Robert D. Gibbons
Illinois State Psychiatric Inst.
Rm 529W
1601 W. Taylor Street
Chicago, IL 60612

Dr. Janice Gifford
University of Massachusetts
School of Education
Amherst, MA 01003

Dr. Robert Glaser
Learning Research
& Development Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15260

Dr. Bert Green
Johns Hopkins University
Department of Psychology
Charles & 34th Street
Baltimore, MD 21218

DORNIER GMBH
P.O. Box 1420
D-7990 Friedrichshafen 1
WEST GERMANY

Dr. Ronald K. Hambleton
University of Massachusetts
Laboratory of Psychometric
and Evaluative Research
Hills South, Room 152
Amherst, MA 01003

Dr. Delwyn Harnisch
University of Illinois
51 Gerty Drive
Champaign, IL 61820

Dr. Grant Henning
Senior Research Scientist
Division of Measurement
Research and Services
Educational Testing Service
Princeton, NJ 08541

Ms. Rebecca Hetter
Navy Personnel R&D Center
Code 63
San Diego, CA 92152-6800

Dr. Paul W. Holland
Educational Testing Service, 21-T
Rosedale Road
Princeton, NJ 08541

Prof. Lutz F. Hornke
Institut fur Psychologie
RWTH Aachen
Jaegerstrasse 17/19
D-5100 Aachen
WEST GERMANY

Dr. Paul Horst
677 G Street, #184
Chula Vista, CA 92010

Mr. Dick Hoshaw
OP-135
Arlington Annex
Room 2834
Washington, DC 20350

Dr. Lloyd Humphreys
University of Illinois
Department of Psychology
603 East Daniel Street
Champaign, IL 61820

Dr. Steven Hunka
3-104 Educ. N.
University of Alberta
Edmonton, Alberta
CANADA T6G 2G5

Dr. Huynh Huynh
College of Education
Univ. of South Carolina
Columbia, SC 29208

Dr. Robert Jannarone
Elec. and Computer Eng. Dept.
University of South Carolina
Columbia, SC 29208

Dr. Douglas H. Jones
Thatcher Jones Associates
P.O. Box 6640
10 Trafalgar Court
Lawrenceville, NJ 08648

Dr. Milton S. Katz
European Science Coordination
Office
U.S. Army Research Institute
Box 65
FPO New York 09510-1500

Prof. John A. Keats
Department of Psychology
University of Newcastle
N.S.W. 2308
AUSTRALIA

Dr. G. Gage Kingsbury
Portland Public Schools
Research and Evaluation Department
501 North Dixon Street
P. O. Box 3107
Portland, OR 97209-3107

Dr. William Koch
Box 7246, Meas. and Eval. Ctr.
University of Texas-Austin
Austin, TX 78703

Dr. James Kraatz
Computer-based Education
Research Laboratory
University of Illinois
Urbana, IL 61801

Dr. Leonard Kroeker
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Dr. Jerry Lehnus
Defense Manpower Data Center
Suite 400
1600 Wilson Blvd
Rosslyn, VA 22209

Dr. Thomas Leonard
University of Wisconsin
Department of Statistics
1210 West Dayton Street
Madison, WI 53705

Dr. Michael Levine
Educational Psychology
210 Education Bldg.
University of Illinois
Champaign, IL 61801

Dr. Charles Lewis
Educational Testing Service
Princeton, NJ 08541-0001

Dr. Robert L. Linn
Campus Box 249
University of Colorado
Boulder, CO 80309-0249

Dr. Robert Lockman
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. Frederic M. Lord
Educational Testing Service
Princeton, NJ 08541

Dr. George B. Macready
Department of Measurement
Statistics & Evaluation
College of Education
University of Maryland
College Park, MD 20742

Dr. Gary Marco
Stop 31-E
Educational Testing Service
Princeton, NJ 08451

Dr. James R. McBride
The Psychological Corporation
1250 Sixth Avenue
San Diego, CA 92101

Dr. Clarence C. McCormick
HQ, USMEPCOM/MEPCT
2500 Green Bay Road
North Chicago, IL 60064

Dr. Robert McKinley
Educational Testing Service
16-T
Princeton, NJ 08541

Dr. James McMichael
Technical Director
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Barbara Means
SRI International
333 Ravenswood Avenue
Menlo Park, CA 94025

Dr. Robert Mislevy
Educational Testing Service
Princeton, NJ 08541

Dr. William Montague
NPRDC Code 13
San Diego, CA 92152-6800

Ms. Kathleen Moreno
Navy Personnel R&D Center
Code 62
San Diego, CA 92152-6800

Headquarters Marine Corps
Code MPI-20
Washington, DC 20380

Dr. W. Alan Nicewander
University of Oklahoma
Department of Psychology
Norman, OK 73071

Deputy Technical Director
NPRDC Code 01A
San Diego, CA 92152-6800

Director, Training Laboratory,
NPRDC (Code 05)
San Diego, CA 92152-6800

Director, Manpower and Personnel
Laboratory,
NPRDC (Code 06)
San Diego, CA 92152-6800

Director, Human Factors
& Organizational Systems Lab,
NPRDC (Code 07)
San Diego, CA 92152-6800

Library, NPRDC
Code P201L
San Diego, CA 92152-6800

Commanding Officer,
Naval Research Laboratory
Code 2627
Washington, DC 20390

Dr. Harold F. O'Neil, Jr.
School of Education - WPH 801
Department of Educational
Psychology & Technology
University of Southern California
Los Angeles, CA 90089-0031

Educational Testing Service/Stocking

Dr. James B. Olsen
WICAT Systems
1875 South State Street
Orem, UT 84058

Office of Naval Research,
Code 1142CS
800 N. Quincy Street
Arlington, VA 22217-5000
(6 Copies)

Office of Naval Research,
Code 125
800 N. Quincy Street
Arlington, VA 22217-5000

Assistant for MPT Research,
Development and Studies
OP 01B7
Washington, DC 20370

Dr. Judith Orasanu
Basic Research Office
Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Dr. Jesse Orlansky
Institute for Defense Analyses
1801 N. Beauregard St.
Alexandria, VA 22311

Dr. Randolph Park
Army Research Institute
5001 Eisenhower Blvd.
Alexandria, VA 22333

Wayne M. Patience
American Council on Education
GED Testing Service, Suite 20
One Dupont Circle, NW
Washington, DC 20036

Dr. James Paulson
Department of Psychology
Portland State University
P.O. Box 751
Portland, OR 97207

Dept. of Administrative Sciences
Code 54
Naval Postgraduate School
Monterey, CA 93943-5026

Department of Operations Research,
Naval Postgraduate School
Monterey, CA 93940

Dr. Mark D. Reckase
ACT
P. O. Box 168
Iowa City, IA 52243

Dr. Malcolm Ree
AFHRL/MOA
Brooks AFB, TX 78235

Dr. Barry Riegelhaupt
HumRRO
1100 South Washington Street
Alexandria, VA 22314

Dr. Carl Ross
CNET-PDCD
Building 90
Great Lakes NTC, IL 60088

Dr. J. Ryan
Department of Education
University of South Carolina
Columbia, SC 29208

Dr. Fumiko Samejima
Department of Psychology
University of Tennessee
310B Austin Peay Bldg.
Knoxville, TN 37916-0900

Mr. Drew Sands
NPRDC Code 62
San Diego, CA 92152-6800

Lowell Schoer
Psychological & Quantitative
Foundations
College of Education
University of Iowa
Iowa City, IA 52242

Dr. Mary Schratz
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Dan Segall
Navy Personnel R&D Center
San Diego, CA 92152

Dr. W. Steve Sellman
OASD(MRA&L)
2B269 The Pentagon
Washington, DC 20301

Dr. Kazuo Shigemasu
7-9-24 Kugenuma-Kaigan
Fujisawa 251
JAPAN

Dr. William Sims
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268

Dr. H. Wallace Sinaiko
Manpower Research
and Advisory Services
Smithsonian Institution
801 North Pitt Street, Suite 120
Alexandria, VA 22314-1713

Dr. Richard E. Snow
School of Education
Stanford University
Stanford, CA 94305

Dr. Richard C. Sorensen
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Paul Speckman
University of Missouri
Department of Statistics
Columbia, MO 65201

Dr. Judy Spray
ACT
P.O. Box 168
Iowa City, IA 52243

Dr. Martha Stocking
Educational Testing Service
Princeton, NJ 08541

Dr. William Stout
University of Illinois
Department of Statistics
101 Illini Hall
725 South Wright St.
Champaign, IL 61820

Dr. Hariharan Swaminathan
Laboratory of Psychometric and
Evaluation Research
School of Education
University of Massachusetts
Amherst, MA 01003

Mr. Brad Sympson
Navy Personnel R&D Center
Code-62
San Diego, CA 92152-6800

Dr. John Tangney
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332-6448

Dr. Kikumi Tatsuoka
CERL
252 Engineering Research
Laboratory
103 S. Mathews Avenue
Urbana, IL 61801

Dr. Maurice Tatsuoka
220 Education Bldg
1310 S. Sixth St.
Champaign, IL 61820

Dr. David Thissen
Department of Psychology
University of Kansas
Lawrence, KS 66044

Mr. Gary Thomasson
University of Illinois
Educational Psychology
Champaign, IL 61820

Dr. Robert Tsutakawa
University of Missouri
Department of Statistics
222 Math. Sciences Bldg.
Columbia, MO 65211

Dr. Ledyard Tucker
University of Illinois
Department of Psychology
603 E. Daniel Street
Champaign, IL 61820

Educational Testing Service/Stocking

Dr. Vern W. Urry
Personnel R&D Center
Office of Personnel Management
1900 E. Street, NW
Washington, DC 20415

Dr. David Vale
Assessment Systems Corp.
2233 University Avenue
Suite 440
St. Paul, MN 55114

Dr. Frank L. Vicino
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Howard Wainer
Educational Testing Service
Princeton, NJ 08541

Dr. Ming-Mei Wang
Lindquist Center
for Measurement
University of Iowa
Iowa City, IA 52242

Dr. Thomas A. Warm
Coast Guard Institute
P. O. Substation 18
Oklahoma City, OK 73169

Dr. Brian Waters
HumRRO
12908 Argyle Circle
Alexandria, VA 22314

Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455-0344

Dr. Ronald A. Weitzman
Box 146
Carmel, CA 93921

Major John Welsh
AFHRL/MOAN
Brooks AFB, TX 78223

Dr. Douglas Wetzel
Code 51
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. Rand R. Wilcox
University of Southern
California
Department of Psychology
Los Angeles, CA 90089-1061

German Military Representative
ATTN: Wolfgang Wildgrube
Streitkraefteamt
D-5300 Bonn 2
4000 Brandywine Street, NW
Washington, DC 20016

Dr. Bruce Williams
Department of Educational
Psychology
University of Illinois
Urbana, IL 61801

Dr. Hilda Wing
NRC MH-176
2101 Constitution Ave.
Washington, DC 20418

Dr. Martin F. Wiskoff
Defense Manpower Data Center
550 Camino El Estero
Suite 200
Monterey, CA 93943-3231

Mr. John H. Wolfe
Navy Personnel R&D Center
San Diego, CA 92152-6800

Dr. George Wong
Biostatistics Laboratory
Memorial Sloan-Kettering
Cancer Center
1275 York Avenue
New York, NY 10021

Dr. Wallace Wulfeck, III
Navy Personnel R&D Center
Code 51
San Diego, CA 92152-6800

Dr. Kentaro Yamamoto
03-T
Educational Testing Service
Rosedale Road
Princeton, NJ 08541

Dr. Wendy Yen
CTB/McGraw Hill
Del Monte Research Park
Monterey, CA 93940

Dr. Joseph L. Young
National Science Foundation
Room 320
1800 G Street, N.W.
Washington, DC 20550

Mr. Anthony R. Zara
National Council of State
Boards of Nursing, Inc.
625 North Michigan Avenue
Suite 1544
Chicago, IL 60611

Dr. Peter Stoloff
Center for Naval Analysis
4401 Ford Avenue
P.O. Box 16268
Alexandria, VA 22302-0268